Basic Considerations for Aseptic Process Design: Non-Newtonian Fluids

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Overview

- What is viscosity? Measurement.
- Newtonian and non-Newtonian fluids
  - Power Law fluids
- We are interested in what is happening in the hold tube
- Laminar and turbulent flow
  - Reynolds Number determination for Newtonian and non-Newtonian fluids
- Fastest fluid element determination
  - Newtonian and non-Newtonian fluids
  - Laminar and Turbulent flow
- Example calculations
- Summary / Conclusions / Recommendations

Objectives, Scope

For Hold Tube determinations, both Newtonian, and in particular, non-Newtonian liquids:

- Discuss flow regimes of liquids flowing in tubes
- Discuss flow profiles and how they effect determination of hold times in a hold tube

For these discussions we will only consider:

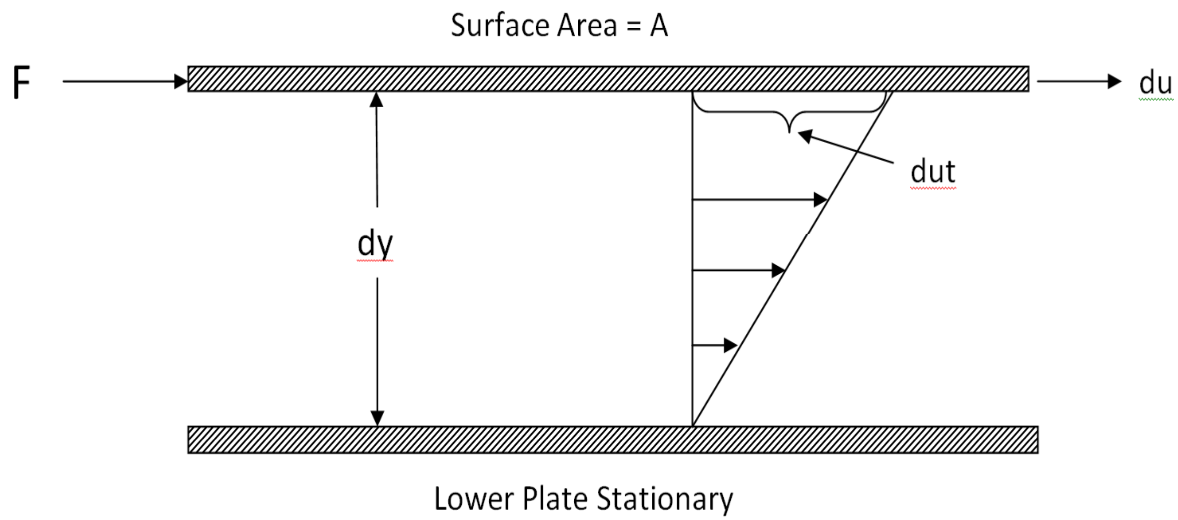
- Liquid flow in smooth, straight, constant diameter tubing
- Fully developed flow profiles
- Emulsions and suspensions are included (typical food products)

We will not consider:

- Changes in diameter, elbows, tees, and other intrusions or fittings
- Presence of particulates or gas
- Flow in coils

Definition of Viscosity

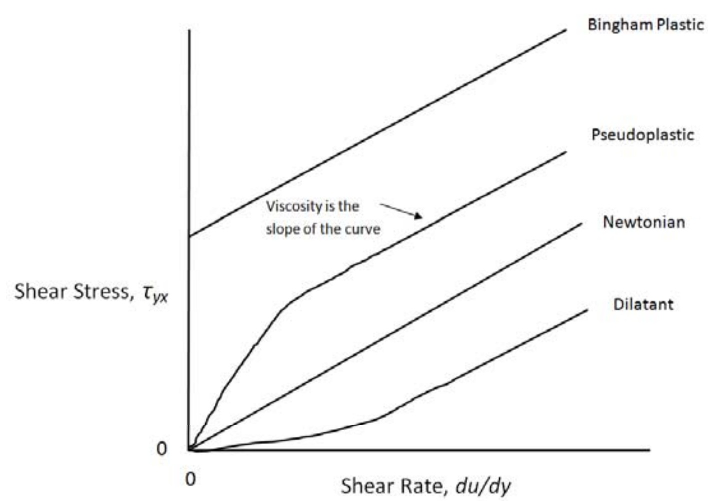
- Viscosity is the resistance to flow of a fluid



- Where:
- Shear Rate = Velocity gradient = du/dy
- Shear Strain is:

$$\tau_{yx} = F / A$$

Definition of Viscosity



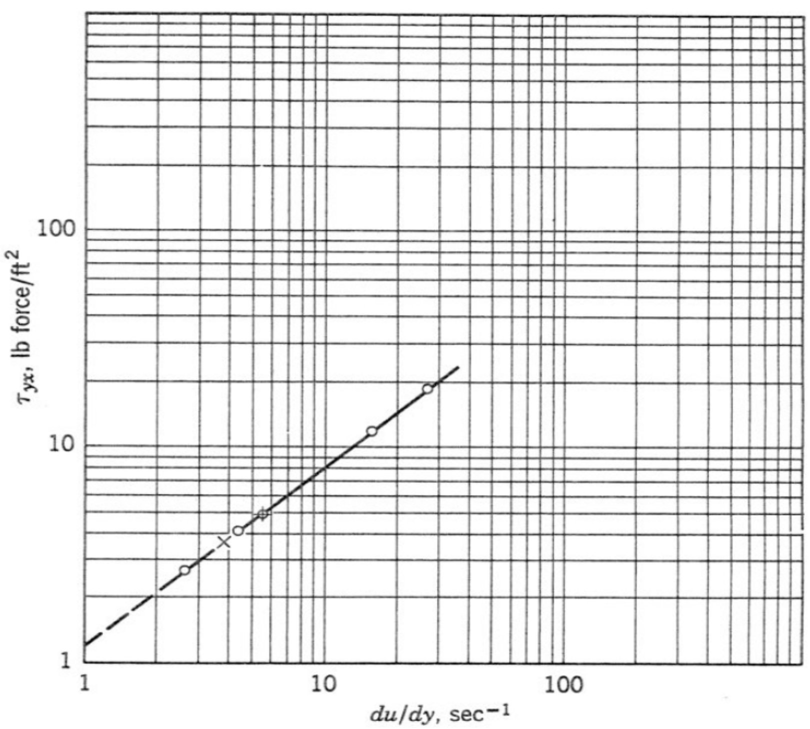
- Viscosity is the slope of Shear Stress vs. Shear rate at any point on the curve. Note that for Newtonian fluids the viscosity is always the same, regardless of shear rate.
- For pseudoplastic (non-Newtonian) fluids, the following results as the power law model:

$$\tau = K\left(\frac{du}{dy}\right)^n$$

- K and n are evaluated on a log-log plot. Note that if n=1, the liquid is Newtonian, and K becomes viscosity.

Viscosity Determination

- State of the art equipment, software, and rheologist!
- Below is log-log plot of shear strain and shear rate at a particular temperature. Note that the data shows that the liquid follows the Power Law very well.



Reference: Skelland, A. Non-Newtonian Flow and Heat Transfer. 1967. John Wiley and Sons, Inc.

Analyzing Flow – Newtonian Liquids

- Flow is either Laminar, or Transition, or Turbulent
- This is assessed by calculating the Newtonian Reynolds Number:

$$Re = \frac{\rho DV}{\mu}$$

Where: ρ is density in lbs. / cu. ft.
D is inside diameter of tube, in ft.
V is bulk average velocity in ft. / s
μ is viscosity in lb./ ft.-s

- From literature, all references agree that for Re <2100, flow is Laminar.
- It is not clear for what Re flow becomes “fully turbulent.”
- To analyze μ, need to know the shear rate. For laminar flow:

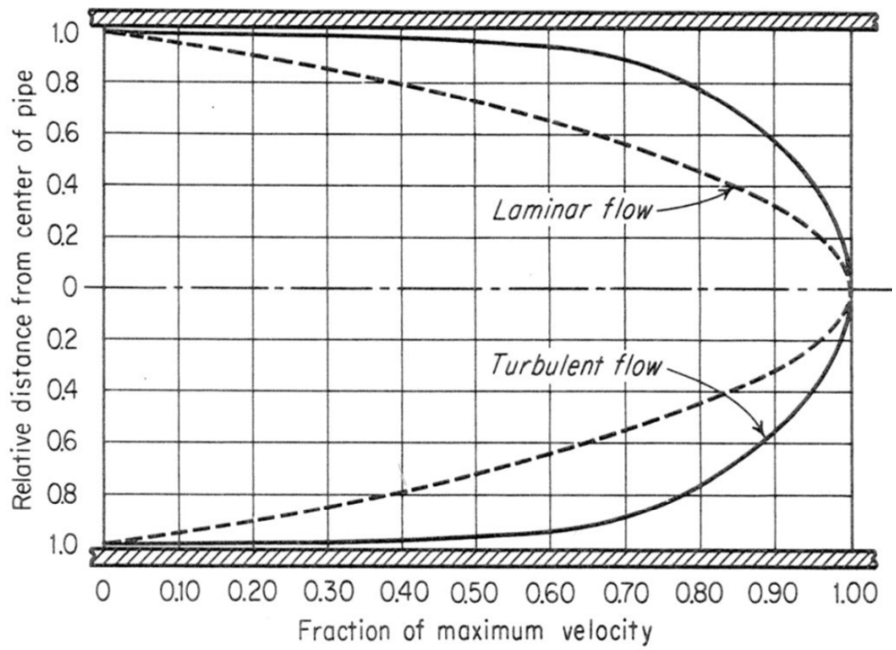
$$(-du \mid dr)_{wall} = \left(\frac{3n+1}{4n}\right)\left(\frac{8V}{D}\right)$$

- For turbulent flow, this is extremely complicated to determine

Flow Profile

- For Laminar flow, for Newtonian liquids, the flow profile is a parabola with maximum velocity along the centerline:

$$\frac{u}{u_{max}} = 1 - \left(\frac{r}{r_w}\right)^2$$



- And further analysis shows that: $$\frac{V_{avg}}{u_{max}} = 0.5$$

Flow profile for Laminar Power Law Liquids

- The flow profile in Laminar flow becomes more blunted as the liquid becomes more and more non-Newtonian (n decreasing).
- Analysis has shown that

$$u = V_{avg}\left(\frac{3n+1}{n+1}\right)\left[1 - \left(\frac{r}{R}\right)^{(n+1)/n}\right]$$

- Or at the centerline (r=0)

$$u_{max} = V_{avg}\left(\frac{3n+1}{n+1}\right)$$

- Which reduces to

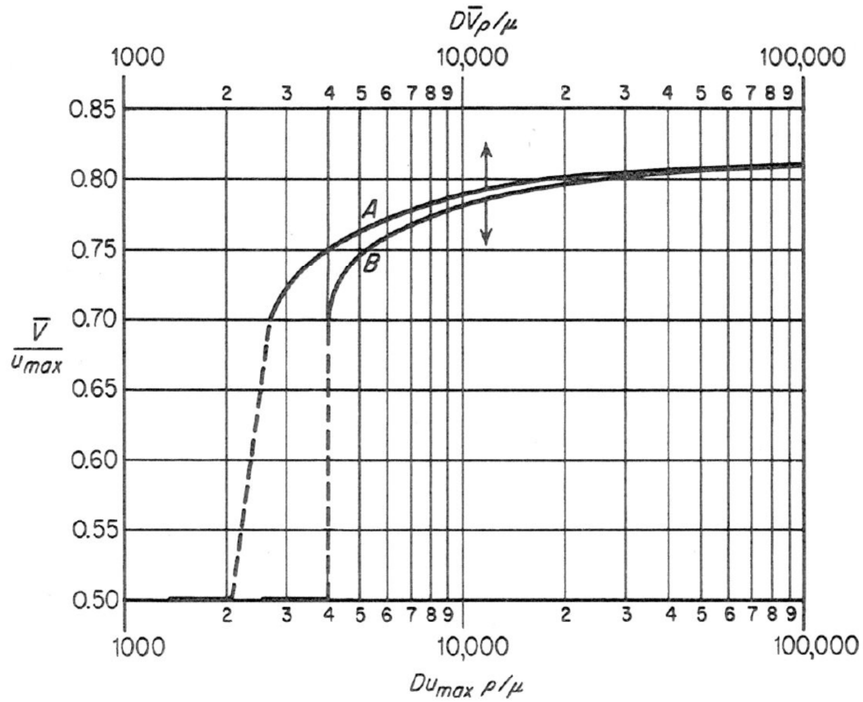
$$\frac{V_{avg}}{u_{max}} = 0.5$$

when n=1. It is generally recognized that for food products, it is very rare for n>1.

Reference: Skelland, A. Non-Newtonian Flow and Heat Transfer. 1967. John Wiley and Sons, Inc.

Flow Profile – Turbulent flow, Newtonian

- We have said that for laminar flow, the fastest fluid element is 2.0 times bulk average velocity. What is it for turbulent flow?
- Many authors have tried to model this (many complicated models that will not be shown here). Some of the models show that u_{max} / V_{avg} to approach 1.2. However, the data found shows what this value should be is not clear.
- And what if the liquid is non-Newtonian?



Reference: McCabe, W.L., and Smith, J.C. Unit Operations in Chemical Engineering. 1967. McGraw Hill Book Company.

More Flow Profile info…

- Another reference. Clearly, based on these, u_{max} / V_{avg} varies with Re; in other words the value appears to vary from ~0.75 to ~ 0.85 as the flow becomes more turbulent. Newtonian liquids.

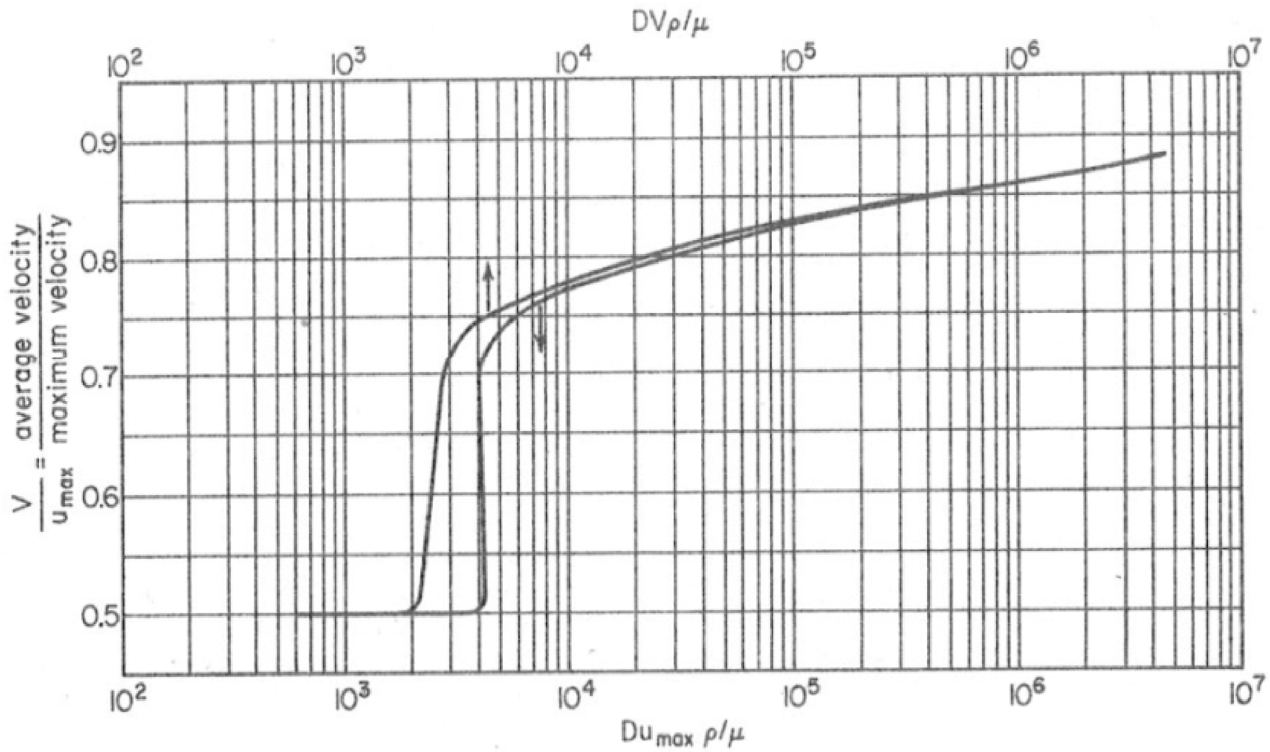
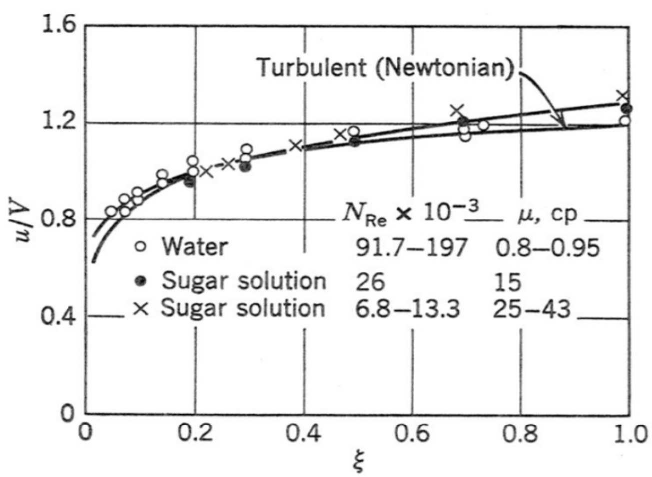
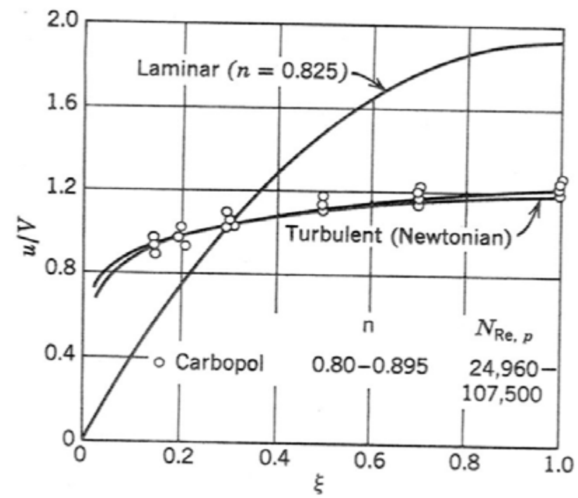
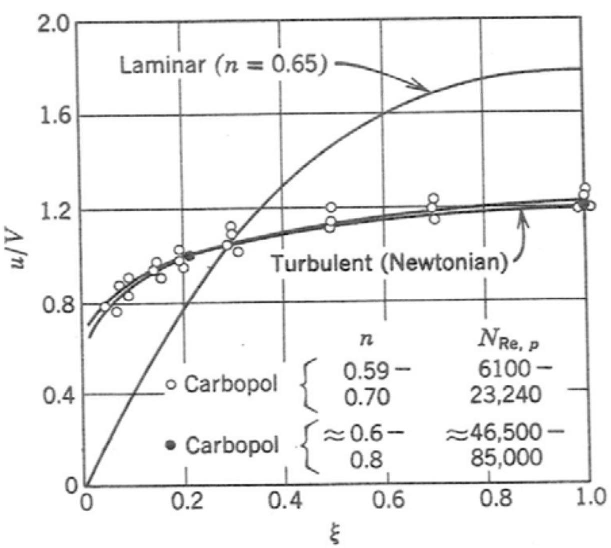


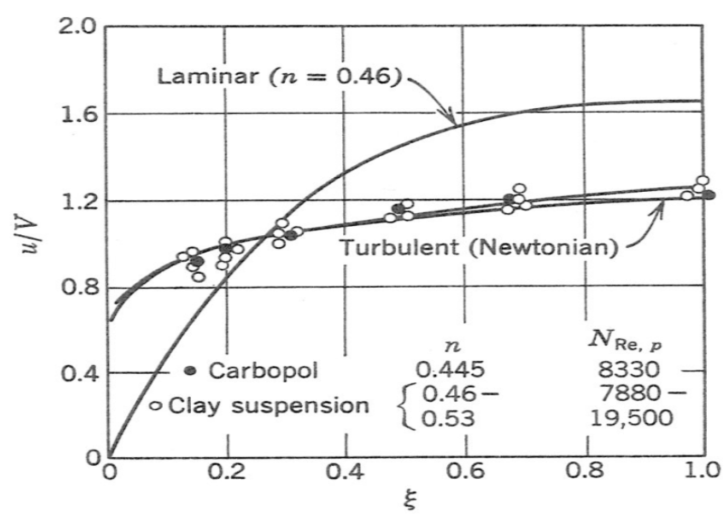
FIG. 5-14 Velocity ratio versus Reynolds number for smooth circular pipes. [*Based on data from Rothfus, Archer, Klimas, and Sikchi,* Am. Inst. Chem. Eng. J., *3, 208 (1957).*]

Various flow profile data









Reference: Skelland, A. Non-Newtonian Flow and Heat Transfer. 1967. John Wiley and Sons, Inc.

Reynolds Number for non-Newtonian Flow

- We have said that for Newtonian flow the Reynolds Number is:

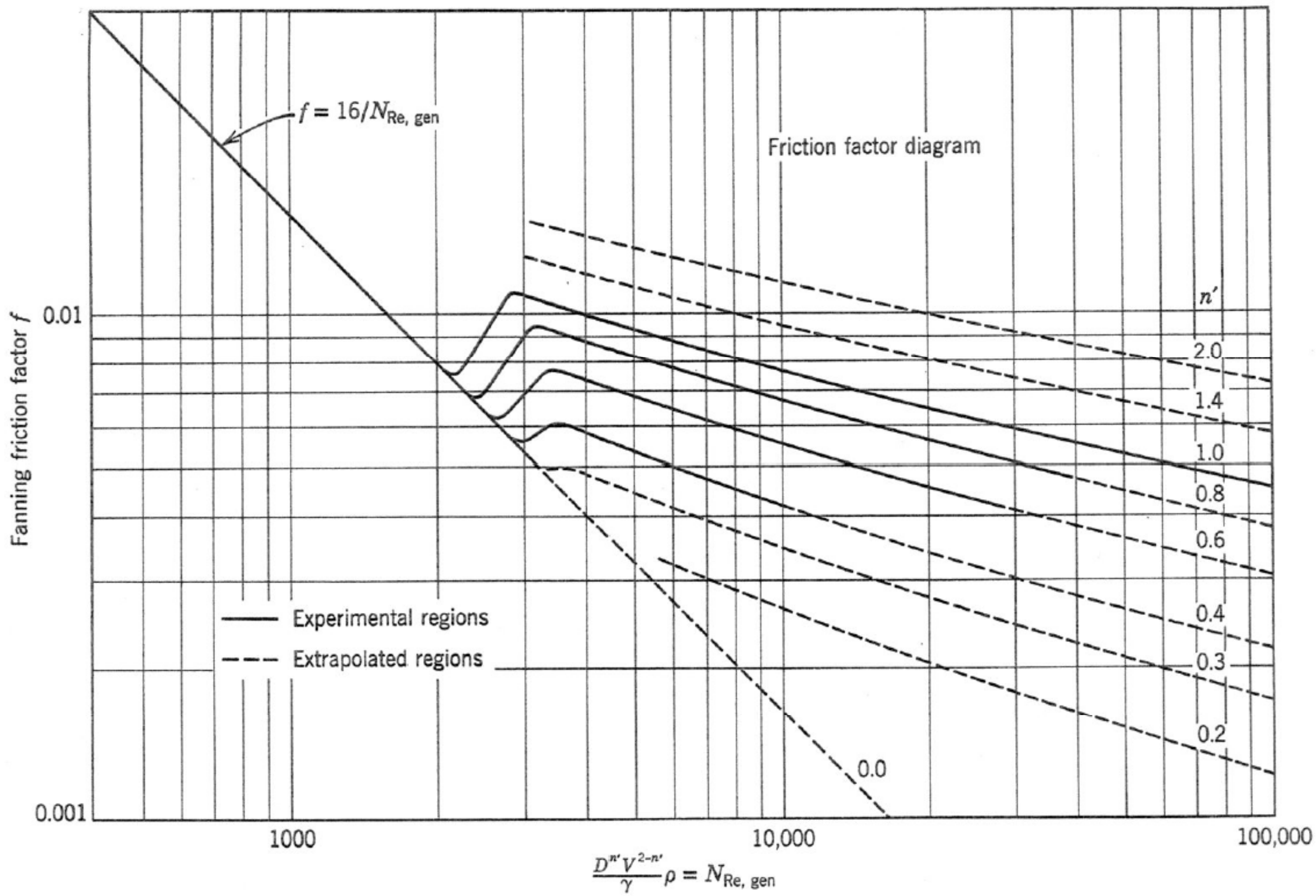
$$Re = \frac{\rho D V}{\mu}$$

- Analysis by various authors has shown that when it is a Power Law liquid, the following Generalized Reynolds Number is to be used:

$$Re_{gen,Power\ Law} = \frac{D^n V^{2-n} \rho}{8^{n-1} K \left(\frac{3n+1}{4n}\right)^n}$$

- Note that if n=1, then the model reverts to the Newtonian Reynolds number shown above, with K becoming the Newtonian viscosity.

When is one truly in the turbulent flow regime?



Reference: Skelland, A. Non-Newtonian Flow and Heat Transfer. 1967. John Wiley and Sons, Inc.

Let's do an exercise!

- Example: actual and extrapolated rheological data from a beverage concentrate (note: data at temp. > 170F were extrapolated).
- Liquid is very non-Newtonian and follows the power law quite well.

Temp., ºF	n	K, Pas^n	Viscosity, mPas@100/s	Viscosity, mPas@500/s
50	0.792	0.272	104.2	74.5
80	0.802	0.128	51.6	37.5
110	0.801	0.080	31.8	23.1
140	0.789	0.057	21.5	15.3
170	0.765	0.045	15.1	10.4
200	0.731	0.037	10.7	6.9
230	0.685	0.032	7.5	4.5
260	0.629	0.028	5.1	2.8
290	0.561	0.026	3.4	1.7

Exercise continued

- Let's work through a number of scenarios for assessing the maximum tube centerline velocity as compared to bulk average
- Most conservative: without any rheology data, nor any calculations, claim laminar flow with

$$\frac{V_{avg}}{u_{max}} = 0.5$$

- Since we have collected actual data up to 170F, we will take n=0.765, and since for laminar flow, power law fluids

$$u_{max} = V_{avg}\left(\frac{3n+1}{n+1}\right)$$

and analyzing,

$$\frac{V_{avg}}{u_{max}} = 0.53$$

Exercise continued

- Let's assume that we actually did collect data up to the hold tube temperature of 290F – then n=0.561, and analyzing

$$\frac{V_{avg}}{u_{max}} = 0.58$$

- The next situation is that you are still not satisfied as this product is extremely temperature-time sensitive, so we do not wish to over process…
- So now we calculate the Power Law Reynolds Number!

$$Re_{gen,Power\ Law} = \frac{D^n V^{2-n} \rho}{8^{n-1} K \left(\frac{3n+1}{4n}\right)^n}$$

Analysis

Nestlé Good Food, Good Life

- We have a UHT system whereby the product hold tube temperature is 290F. It is an indirect system, and we know that the flow rate in the hold tube is 40 gpm; the hold tube is 2 inch sanitary tubing (1.87 inch ID), and the density is 68.8 lb/ft^3
- Since we only have actual data up to 170F, we will use the rheology info for that temperature.
- Substituting into the Power Law Reynolds number (be very careful of units!) gives us Re= 5301, meaning $u_{max} / V_{avg} = \sim 0.75$
- What if we really did have rheology data taken up to the 290F hold tube temperature? This would give us (only changing n and K), using the Power Law Reynolds Number, Re= 27,500, meaning $u_{max} / V_{avg} = \sim 0.81$

- Note that for this analysis we do not need to know the viscosity!

More Analysis

- Now isn't using the Power Law Re too complicated? Let's try and simplify things and use the Newtonian Re.
- So, now we need to find the viscosity at 170F (highest temperature we actually have data for).
- And this is…..? Maybe not so simple. Do we use the viscosity at a shear rate of 100/s?
- OK, then what is the shear rate for the fluid flowing through the tube. This gets very, very complicated, so let's assume the viscosity is 15.1 cP. At this viscosity, Re calculates to 4720 with u_{max} / V_{avg} = ~0.75
- This actually comes in lower by ~ 10%. Thus, working backwards, the viscosity @100/s shear rate of 15.1 cP was actually ~ 10% too high (on the conservative side).
- We can thus say that the shear rate in the pipe was actually a little more than 100/s.

Further Analysis

- So, you are now all wondering, to keep things simple, can we simply calculate a shear rate, then get a viscosity, and then use the Newtonian Re?
- Calculating the Turbulent flow shear rate is very complicated, and very iterative.
- Suggest to use the Laminar flow shear rate as an *approximation*, and then conservatively estimate a viscosity and then use the simpler Re calculation.

$$(-du \mid dr)_{wall} = \left(\frac{3n+1}{4n}\right)\left(\frac{8V}{D}\right)$$

- In our example, this comes to a shear rate of 260/s. Using this and then conservatively estimating viscosity would give us a cP = ~13 to 15 to use. This would then give us a fair Re estimate.

So….

So, what does all this potentially mean?

- Let’s say I have a laminar flow situation, but from data an n= 0.6. Then, $u_{max} / V_{avg} = 0.57$.
- This is an increase in lethality of 14%

Example:

- Hold Tube temperature of 288°F with a hold time ($u_{max} / V_{avg} = 0.5$) of 0.10 minutes.
- With Tr= 250°F, and z=18°F, F_o=12.9. For $u_{max} / V_{avg} = 0.57$, this becomes F_o=14.7.
- To keep the same F_o=12.9, we could use a hold tube temperature of almost exactly1°F less (287 instead of 288°F).

- Note that this is a small change. But, whether the flow is laminar or turbulent is not a small difference.

Summary and Conclusions

- Flow in a tube is characterized as laminar, transition, or turbulent.
- The generally recognized method of assessing this is by calculating the Reynolds number.
- The flow regime is effected by the rheology of the fluid. Food products can be Newtonian or non-Newtonian. Most foods that are non-Newtonian can be modeled as Power Law fluids.
- A general survey of the literature has been made as it relates to hold tube calculations in relation to estimating V_{avg} / u_{max}
- All agree that V_{avg} / u_{max} for laminar flow =0.5 (a parabola) *for Newtonian liquids.*
- Laminar flow in all cases is when Re < 2100. Above this, flow may be laminar, or may be in a transition regime, until it becomes turbulent.

Summary and Conclusions

- The end of the transition regime appears to be between a Re of 2100 to ~ 4000 plus, and depends on the Power Law n index.
- If the liquid has rheology that follows the Power Law, then the flow profile in Laminar flow is:

$$u_{max} = V_{avg}\left(\frac{3n+1}{n+1}\right)$$

- As the flow regime changes from laminar, to transition, and then into turbulent, the flow profile flattens as the degree of turbulence increases. In the turbulent regime, V_{avg} / u_{max} does not appear to be characterized by a single value, unlike laminar flow (for Newtonian).
- It is not fully clear what values should be used for V_{avg} / u_{max} for turbulent flow as it does appear to be a function of Re.
- V_{avg} / u_{max} for non-Newtonian liquids is virtually identical to that for Newtonian liquids
- Newtonian liquids (n=1) can be viewed as a special case of non-Newtonian liquids, rather than the other way around.

Summary and Conclusions

- To be the most accurate, and if one has good rheology data and if the liquid has properties that follow the Power law, then the Power Law Reynolds Number can be used to estimate Re (do not need to know the viscosity!).
- An alternative is to estimate the shear rate in the tube using the expression for laminar flow, and from that conservatively estimate the viscosity, and then use the Newtonian Re estimation.

Recommendations

- The use of V_{avg} / u_{max} values other than 0.5 for laminar flow as long as it is justified (good rheology data!).
- The use of V_{avg} / u_{max} =0.83 value in all cases for turbulent flow can be debated, and the value of 0.83 is not necessarily in the conservative direction.
- Key is to correctly assess if flow is truly turbulent