

# ON THE ROUTE TO OPTIMIZE BATCH THERMAL PROCESSING



*Ricardo Simpson and Alik Abakarov*

Universidad Santa María

CHILE

# OUTLINE

---

- Introduction
- Background
- Advances in thermal process optimization
- **Plant design optimization**
- **Optimizing processing plant productivity**
- A mixed integer linear programming model to optimize canned food plants operation
- Study case
- Conclusions and Future Trends

# INTRODUCTION

---

Optimization of thermal processing is of great interest, because the canning industry plays and will play an important role within the economy of the food processing sector.

According to Oliveira, 2003 optimizing processing plant productivity, although a simple proposition, has received little attention and there is no paper really looking for a comprehensive business analysis involving thermal process optimization.

# INTRODUCTION

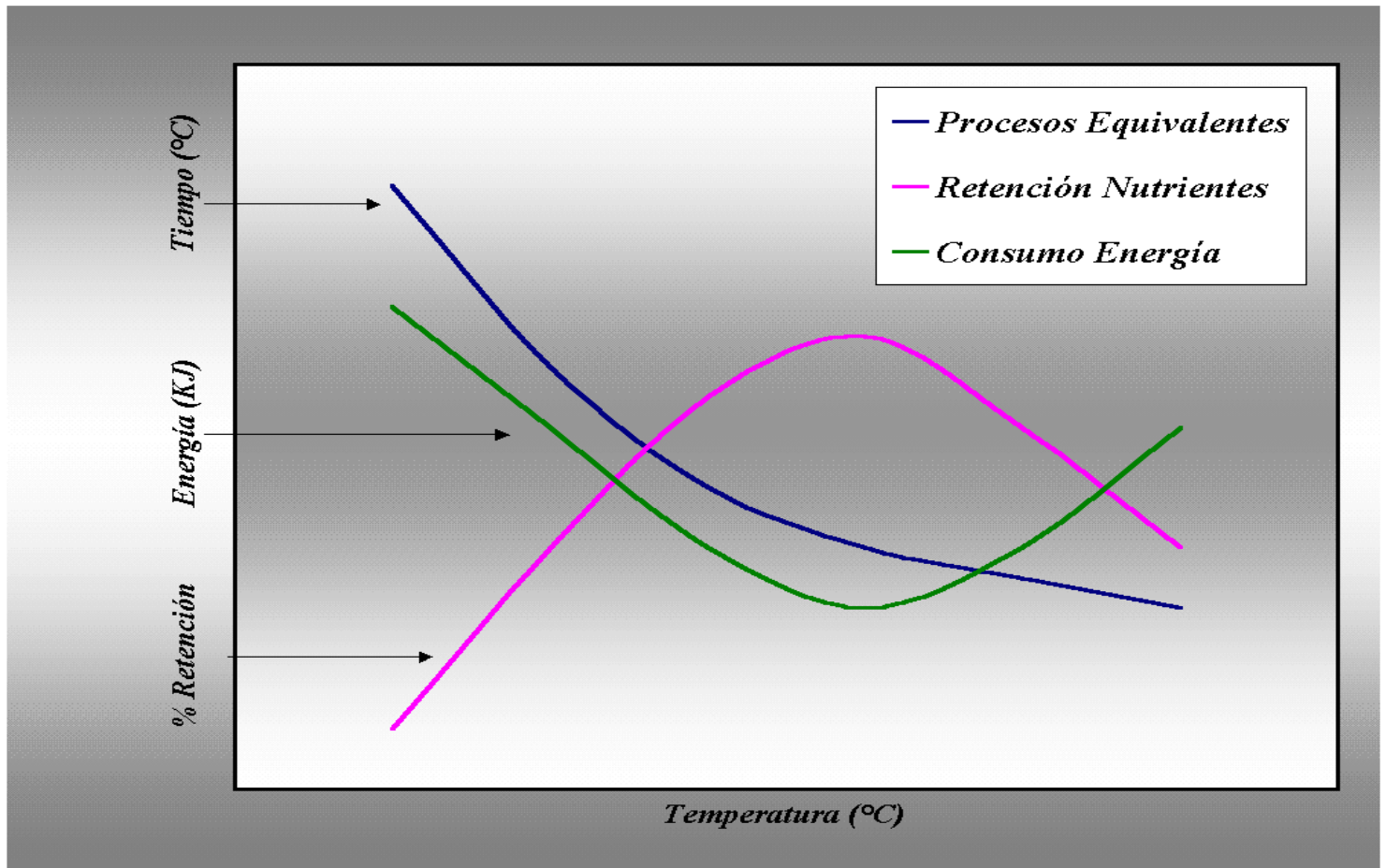
---

Although the literature in food science and thermal processing is very extensive, most the references deal with the microbiological and biochemical aspects of the process or with engineering analysis **of a single unit process operation**, and rarely analyse the operation processing in the context of, manufacturing efficiency.



# INTRODUCTION

We can briefly summarize these findings in the following graph.



# INTRODUCTION

---

We can formulate the following questions:

- a) What is the optimum process for a specific product?
- b) What is the optimum number of autoclaves?
- c) Which is the way to operate the whole manufacturing process?
- d) Is question a) a subset of question c)?

Scientific literature is abundant in relation to the first question but scarce regarding in the second and third questions.

# INTRODUCTION

---

## What is the optimum process for a specific product?

Theory and experience have shown that the best process for a specific product is one with high quality retention, sufficiently low energy consumption, and with the minimum possible processing time.

S. Almonacid, R. Simpson, J.A. Torres. 1993. Time-variable retort temperature profiles for cylindrical cans: batch process time, energy consumption, and quality retention model. *J. Food Proc. Eng.* 16 (4), 171-187.

# INTRODUCTION

---

Which is the best way to operate the whole manufacturing process?

Not only is this question unanswered, but also, some important information is missing, or hard to find for a comprehensive answer.

Some attempts to answer this question are described in this presentation.

Simpson, R, Almonacid, S., and Teixeira, A. 2003. *optimization criteria for batch retort battery design and operation in food canning-plants* . Journal of Food Process Engineering, 25(6), 515-538.



# INTRODUCTION

---

## New and needed information

A theoretical approach of simultaneous sterilization is fully described in the scientific literature, and might be, an interesting contribution in the route to optimize canning operation.

A needed information will be the knowledge of transient energy consumption per each batch process.

# INTRODUCTION

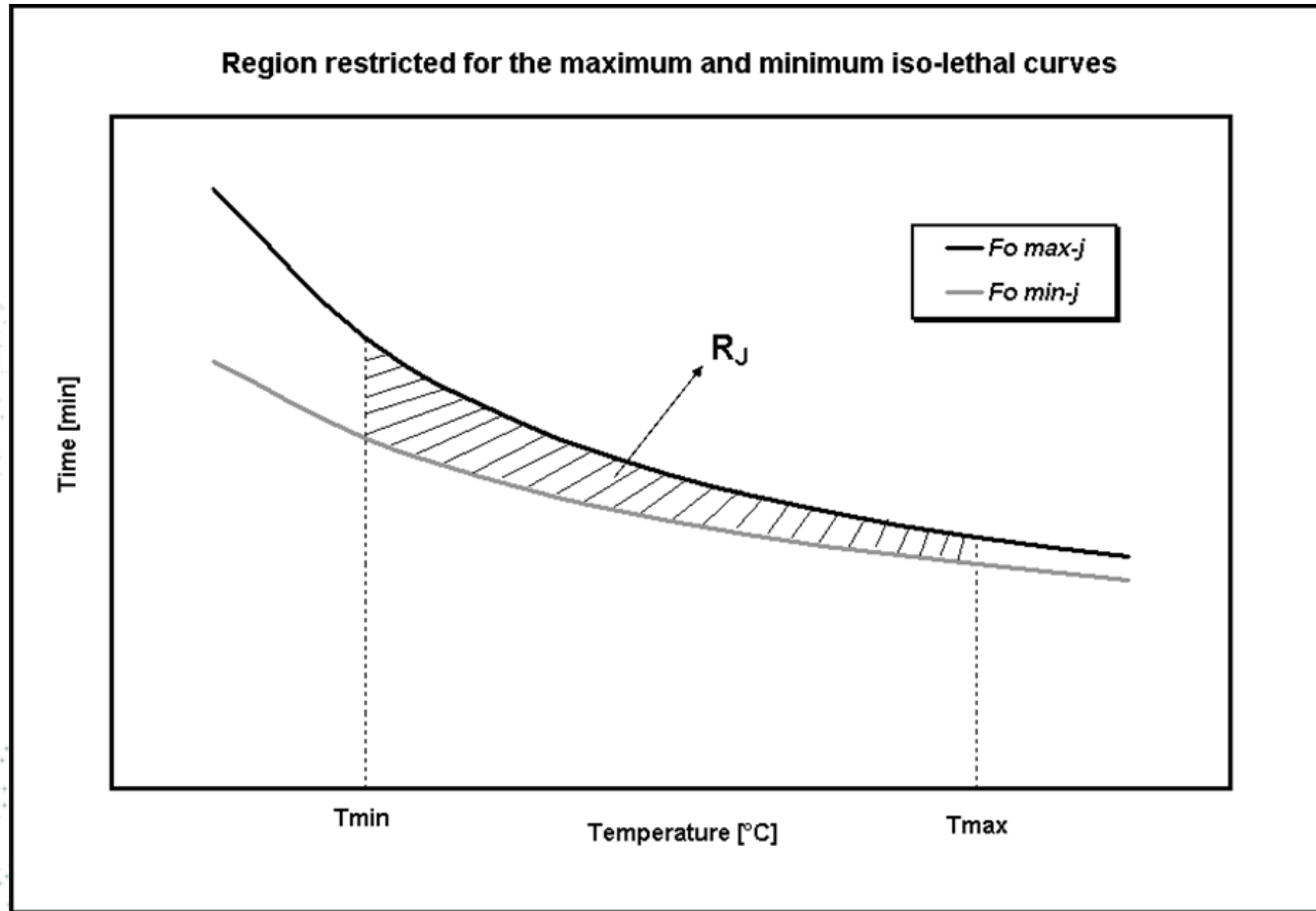
---

## Simultaneous Processing of Different Product Lots in The Same Retort

This optimization criterion mainly applies to the case of small canneries with few retorts that are frequently required to process small lots of different products in various container sizes that normally require different process times and retort temperatures.

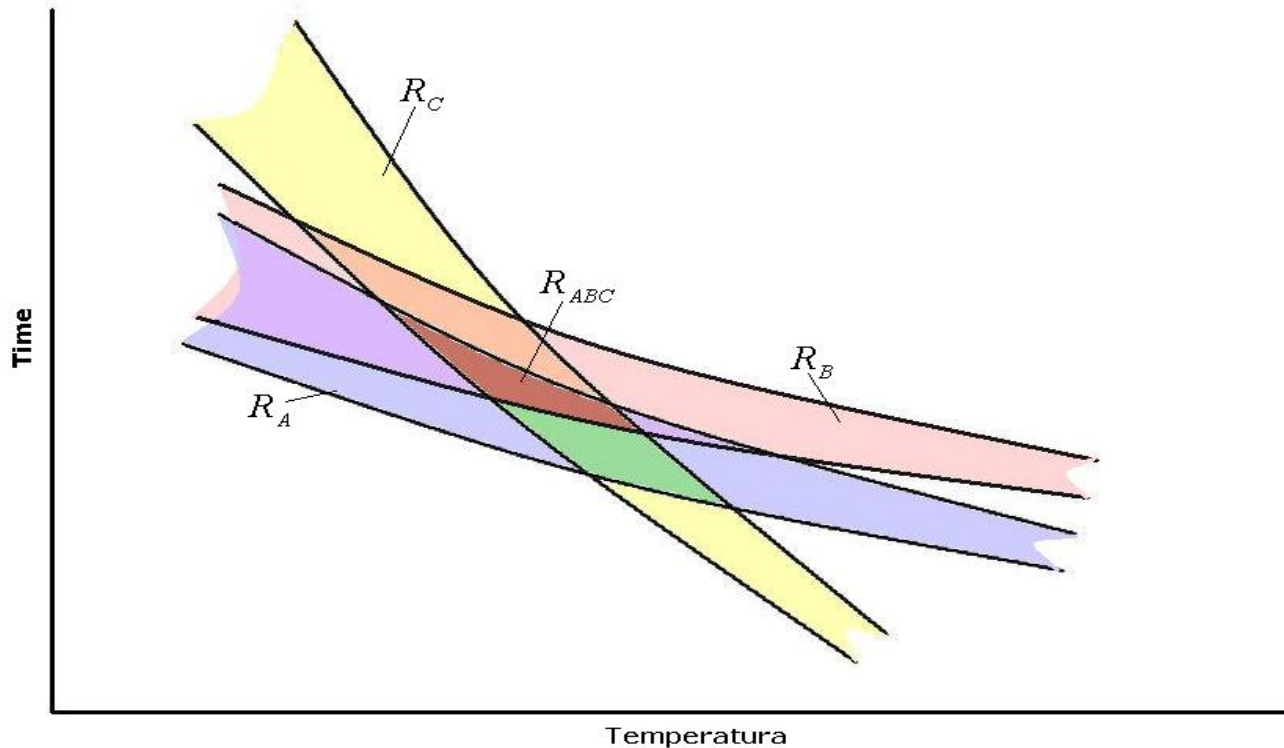
Simpson, R. 2005. Generation of isolethal processes and implementation of simultaneous sterilization utilizing the revisited general method. *Journal of Food Engineering*, Volume 67, Issues 1-2, Pages 71-79.

# INTRODUCTION



# INTRODUCTION

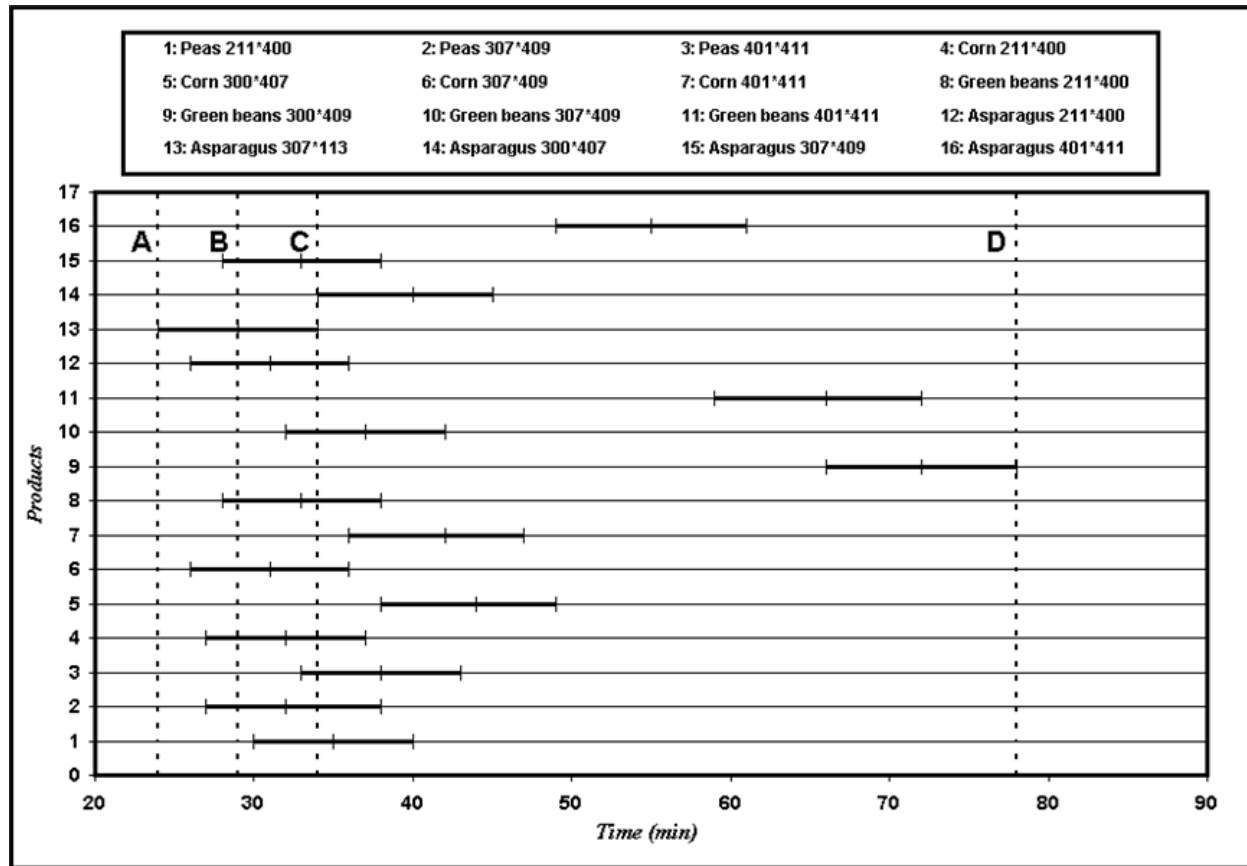
The intersection  $R_{ABC}$  of regions  $R_A$ ,  $R_B$ ,  $R_C$  gives isolethal processes which allow to sterilize three different products A, B and C simultaneously.



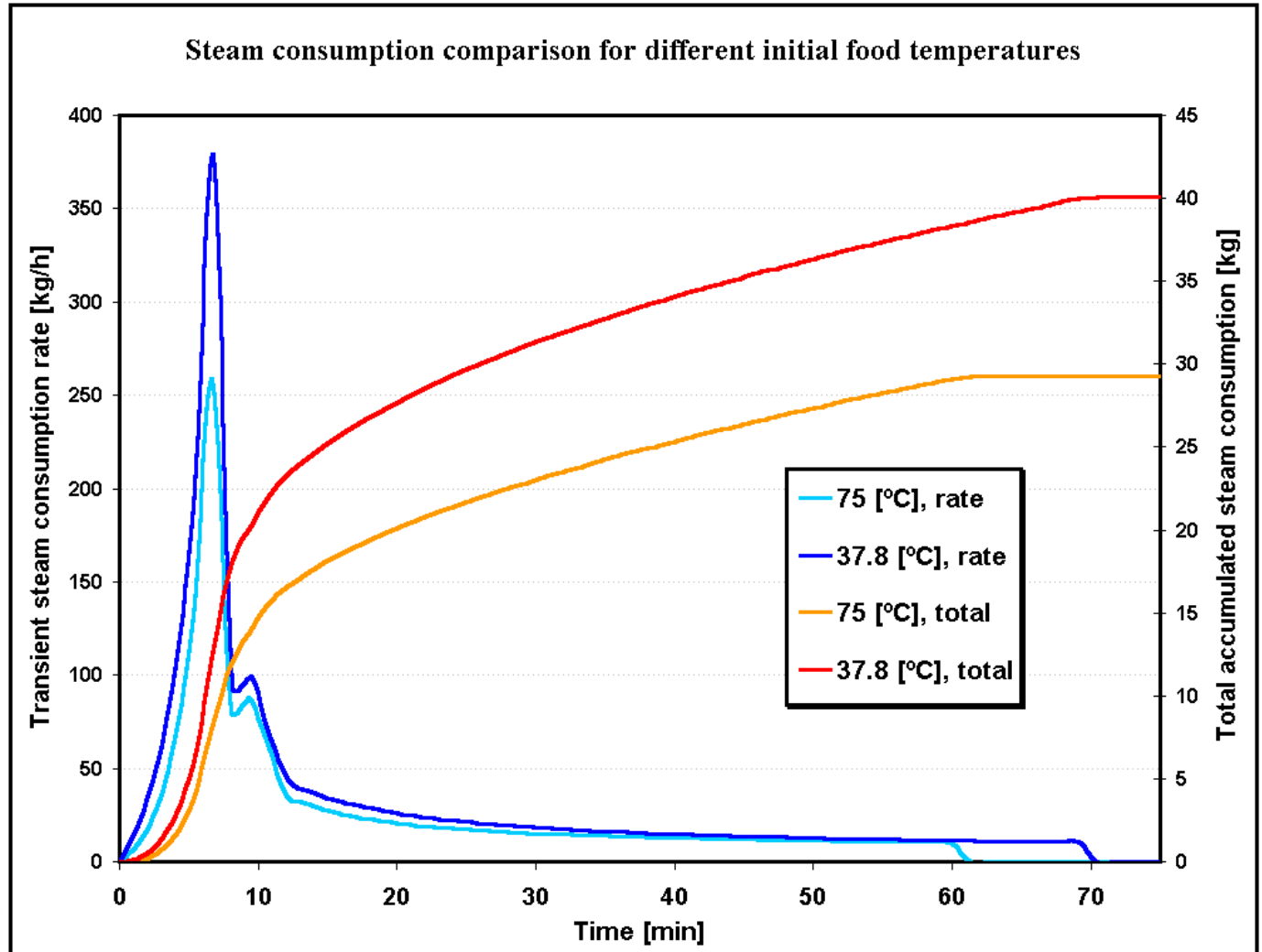


# INTRODUCTION

Process time per each product at different  $F_0$  values (6, 8, and 10 minutes) at TRT = 117 °C



# Transient energy consumption (steam retort)



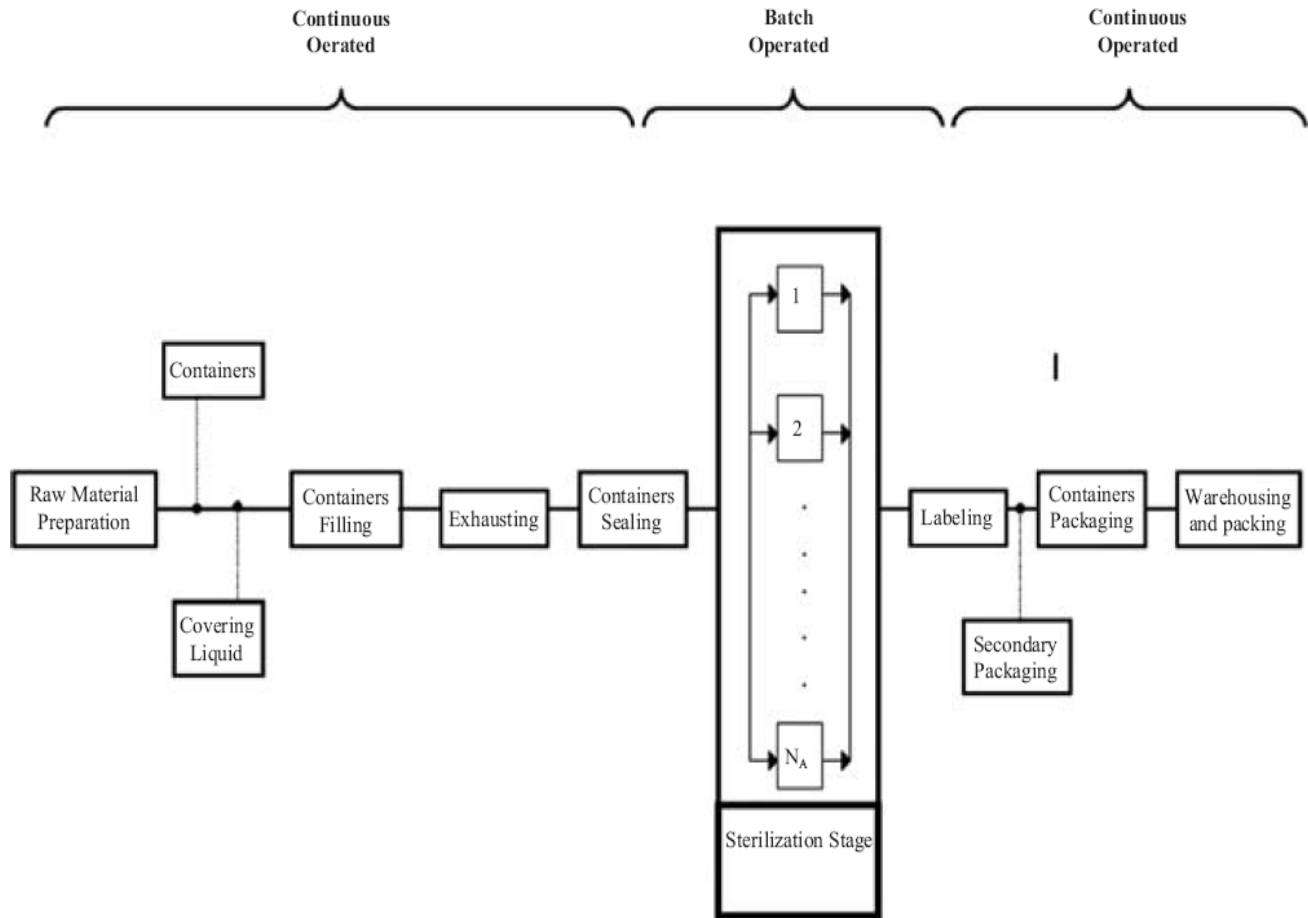
# BACKGROUND

---

Batch processing has been widely practiced but little analyzed in the context of canned food plants.

Since the entire process line operates continuously, food canneries are often overlooked as batch process industries.

# SCHEMATIC REPRESENTATION OF CANNING PROCESS





# BACKGROUND

---

Scheduling to maximize efficiency of batch processing plants has become well known, and it is commonly practiced in **many process industries other than food processing sector.**

Specific application to canning plants has not been addressed in the food process engineering literature.

# ADVANCES IN THERMAL PROCESS OPTIMIZATION

---

Our research group has focussed their efforts in two different aspects:

- a) **How to do the best investment for a new plant?,**
- b) **How to operate autoclaves (scheduling) in a plant under operation (fixed capacity)?**

Both questions will be addressed according to the maximization of the *Net Present Value*.

# PLANT DESIGN OPTIMIZATION

---

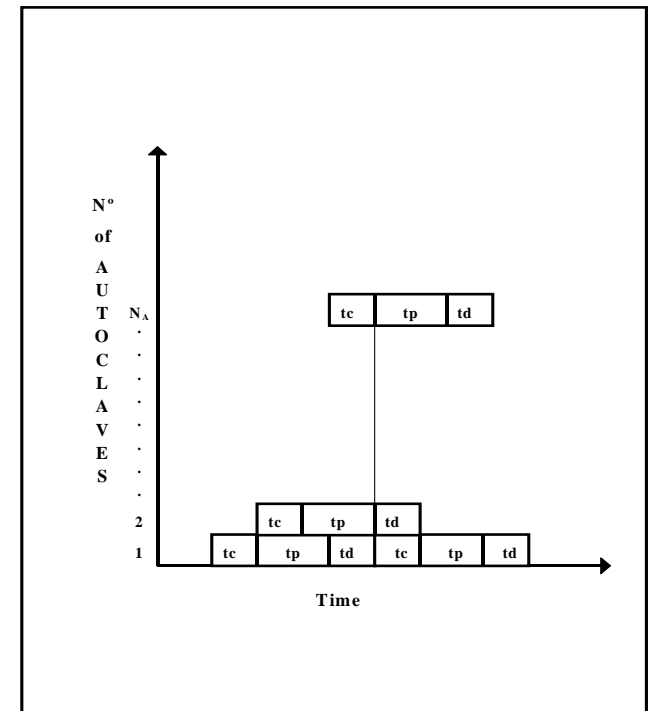
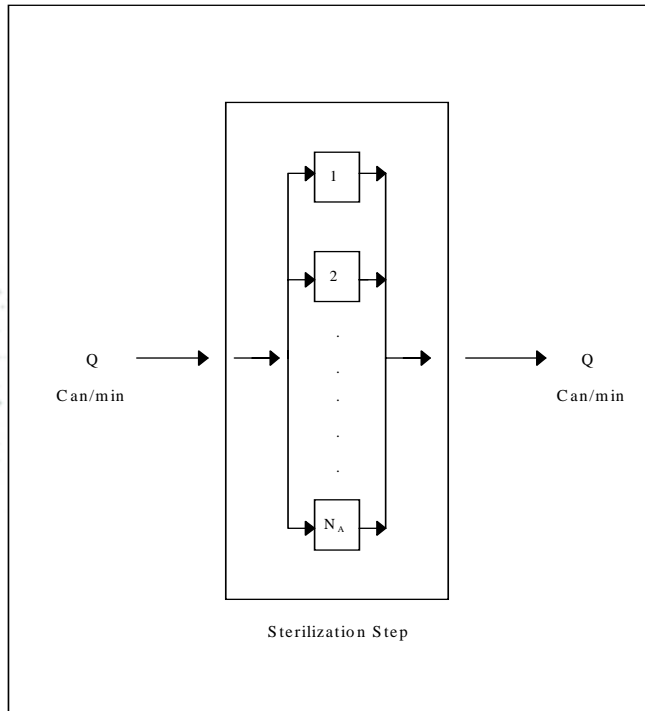
## How to do the best investment for a new plant?

Our group in 2003 published a paper entitled: *Optimization criteria for batch retort battery design and operation in food canning-plants* (Journal of Food Process Engineering, 25(6), 515-538).

In the referred paper we followed the hierarchical approach proposed by Douglas, 1988

# PLANT DESIGN OPTIMIZATION

Our idea was to have a continuous operation of autoclaves.



This relationship can be expressed mathematically:

$$t_c + t_p + t_d = t_c N_A$$

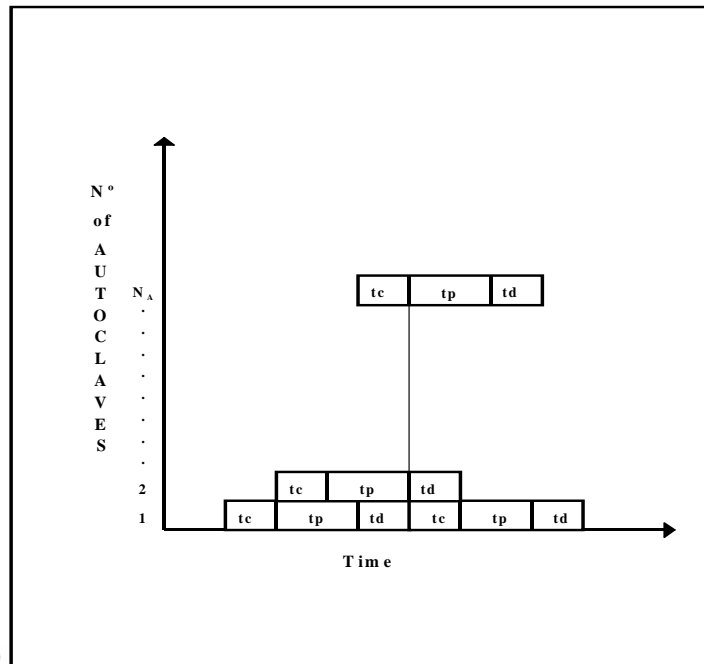


# PLANT DESIGN OPTIMIZATION

Considering loading and unloading times equal ( $t_c = t_d$ ), we get:

$$N_A = 2 + \frac{t_p}{t_c}$$

The following mathematical relationships can relate the plant production capacity (Q) to loading time and retort size:



$$Q t_c = K V$$

$$Q = \frac{K V (N_A - 2)}{t_p}$$

According to Guthrie (1969) the cost of equipment (retorts, etc.) could be expressed as being in proportion to its capacity.

$$C_R = k_R V^a \quad \text{Where exponent } a \text{ is less than one}$$

Therefore the required investment in  $N_A$  retorts, could be expressed as:

$$I_N = N_A (k_R V^a)$$

$$I_N = N_A \left[ k_R \left( \frac{Qt_p}{(N_A - 2)K} \right)^a \right]$$

$$N.P.V. = -N_A \left[ k_R \left( \frac{Qt_p}{(N_A - 2)K} \right)^a \right] - I_B - I_F - I_X + K' \frac{K V (N_A - 2)}{t_p} t_y (P_u - C_u)$$

By inspection

$$\left( \frac{\partial(V(N_A - 2))}{\partial N_A} \right)_{t_p} = 0 \quad \text{then} \quad \frac{d(NPV)}{dN_A} = - \frac{dI(N_A, B, F, x)}{dN_A}$$

As a starting point, a simplified situation will be analysed in which the investment for a specified plant size is affected, only, by the number of retorts ( $N_A$ ).

$$\frac{d(N.P.V.)}{dN_A} = -k_R \left( \frac{Qt_p}{(N_A - 2)K} \right)^a + k_R a N_A \left( \frac{Qt_p}{(N_A - 2)^2 K} \right) \left( \frac{Qt_p}{(N_A - 2)K} \right)^{a-1} = 0$$

According to Guthrie (1969) the exponent  $a$  has a value of 0.6 for horizontal retorts and 0.65 for vertical retorts. Therefore:

$$N_A^* = \frac{2}{1-a}$$

$$N_{AH}^* = \frac{2}{1-0.6} = 5$$

$$N_{AV}^* = \frac{2}{1-0.65} \cong 6$$

# OPTIMIZING PROCESSING PLANT PRODUCTIVITY

---

How to operate the autoclaves (scheduling) in a plant under operation (fixed capacity)?

The objective of this research was to solve the optimal scheduling sterilization problem.

First we carried out a microeconomic analysis to determine an adequate objective function.

# OPTIMIZING PROCESSING PLANT PRODUCTIVITY

---

## Microeconomics analysis

A criterion to optimize process design is to determine the processing conditions that will maximize the Net Present Value ( $NPV$ ) of the invested capital of the process line.

$$N.P.V. = -I + \sum_{j=1}^n \frac{\beta_j}{(1+i)^j} \quad (1)$$



# OPTIMIZING PROCESSING PLANT PRODUCTIVITY

---

The investment will be fixed for a specific plant, and annual benefits are related to the unit-price of the product ( $P_u$ ), costs per unit ( $C_u$ ), and annual production ( $Q^*$ ), as indicated in the following equation:

$$\beta_j = Q_j^*(P_u - C_u) \quad (2)$$

# OPTIMIZING PROCESSING PLANT PRODUCTIVITY

---

Unit-price ( $P_u$ ) is not constant and it is directly related to the final product quality, which will be considered to be constant. In addition, the difference in energy consumption for equivalent processes is not relevant. On the other hand, annual production is strongly influenced by operating conditions and scheduling.

Therefore equation (1) can be expressed as a function of operating conditions and scheduling, as follows:

# OPTIMIZING PROCESSING PLANT PRODUCTIVITY

$$N.P.V. = -I + \sum_{j=1}^n \frac{Q_j^*(OC) * (P_u - C_u)}{(1+i)^j} \quad (3)$$

To find the critical operating conditions and scheduling for maximum  $NPV$ , it is necessary to differentiate equation (3) and then equate it to zero.

$$\frac{d(NPV)}{d(OC)} = -\frac{d(I)}{d(OC)} + \frac{d}{d(OC)} \left( \sum_{j=1}^n \frac{Q_j^*(OC) * (P_u - C_u)}{(1+i)^j} \right) \quad (4)$$

$$\text{Where} \quad \frac{d(I)}{d(OC)} = 0 \quad (5)$$

# OPTIMIZING PROCESSING PLANT PRODUCTIVITY

Then, from equations (4) and (5):

$$\frac{d(NPV)}{d(OC)} = k * (P_u - C_u) \frac{d(Q^*(OC))}{d(OC)} \quad (6)$$

By inspection of equation (5), it is clear that the maximum  $NPV$  can be reached by **minimizing plant operation time.**

# MIXED INTEGER LINEAR PROGRAMMING (MILP) MODEL

Any simultaneous sterilization possibility can be presented as the following triple:

$$\langle v, t, T \rangle$$

where  $v = (v_1, v_2, \dots, v_n)$  is called a simultaneous sterilization vector,  $n$  is number of different product,  $v_j \in \{0, 1\}$ ,  $j \in 1:n$ , and

$$v_j = \begin{cases} 1, & \text{if product } k \text{ can be sterilized,} \\ 0, & \text{otherwise} \end{cases},$$

and  $t, T$  are the necessary time and retort temperature, respectively, used to process a subset of products determined by the vector.



# MIXED INTEGER LINEAR PROGRAMMING (MILP) MODEL

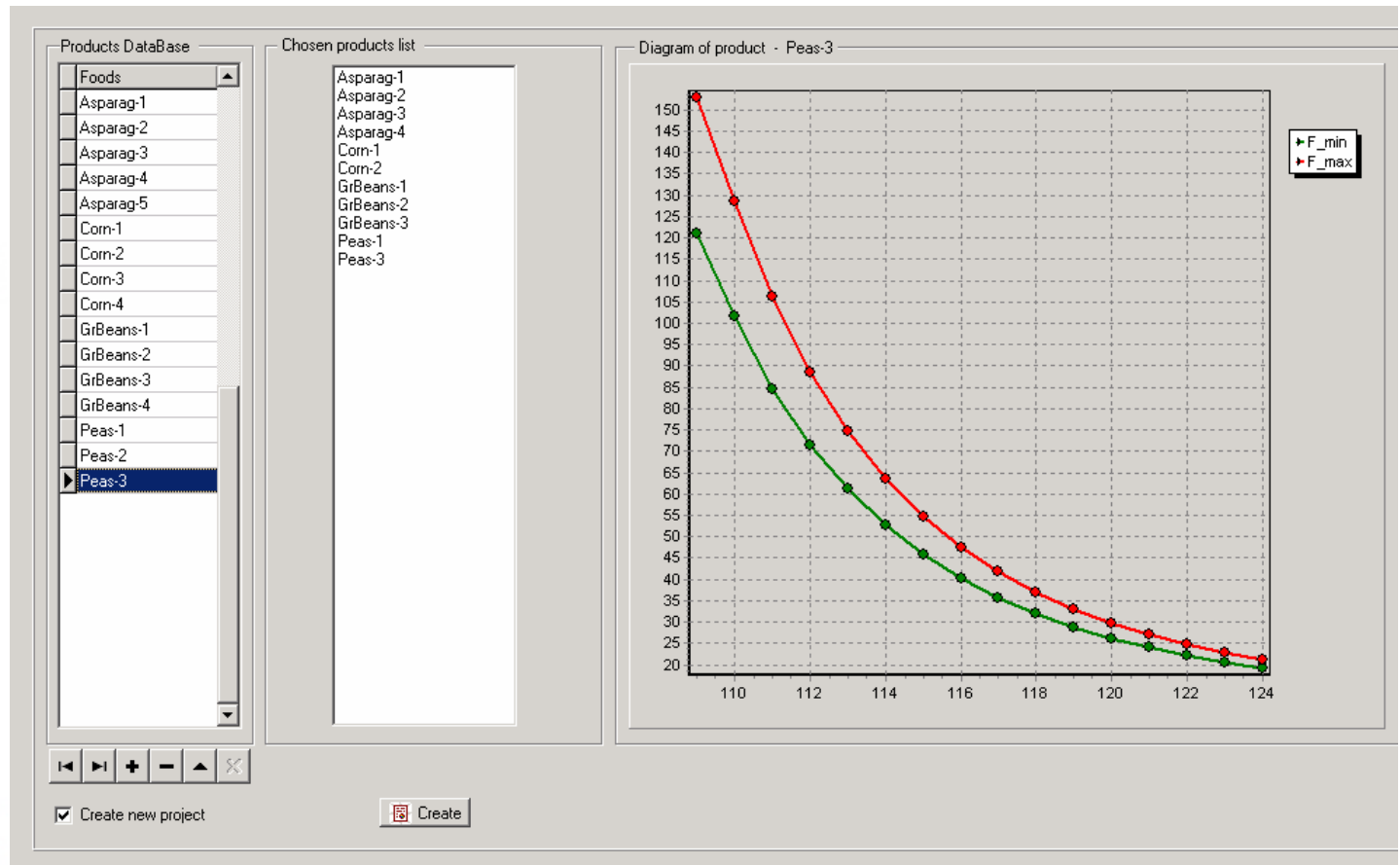
---

The mathematical procedure to find out all possible simultaneous sterilization vectors for a given number of products has been detailed in Simpson (2005).

Simpson, R. 2005. Generation of isolethal processes and implementation of simultaneous sterilization utilizing the revisited general method. *Journal of Food Engineering*, 67, Issues 1-2, 71-79.

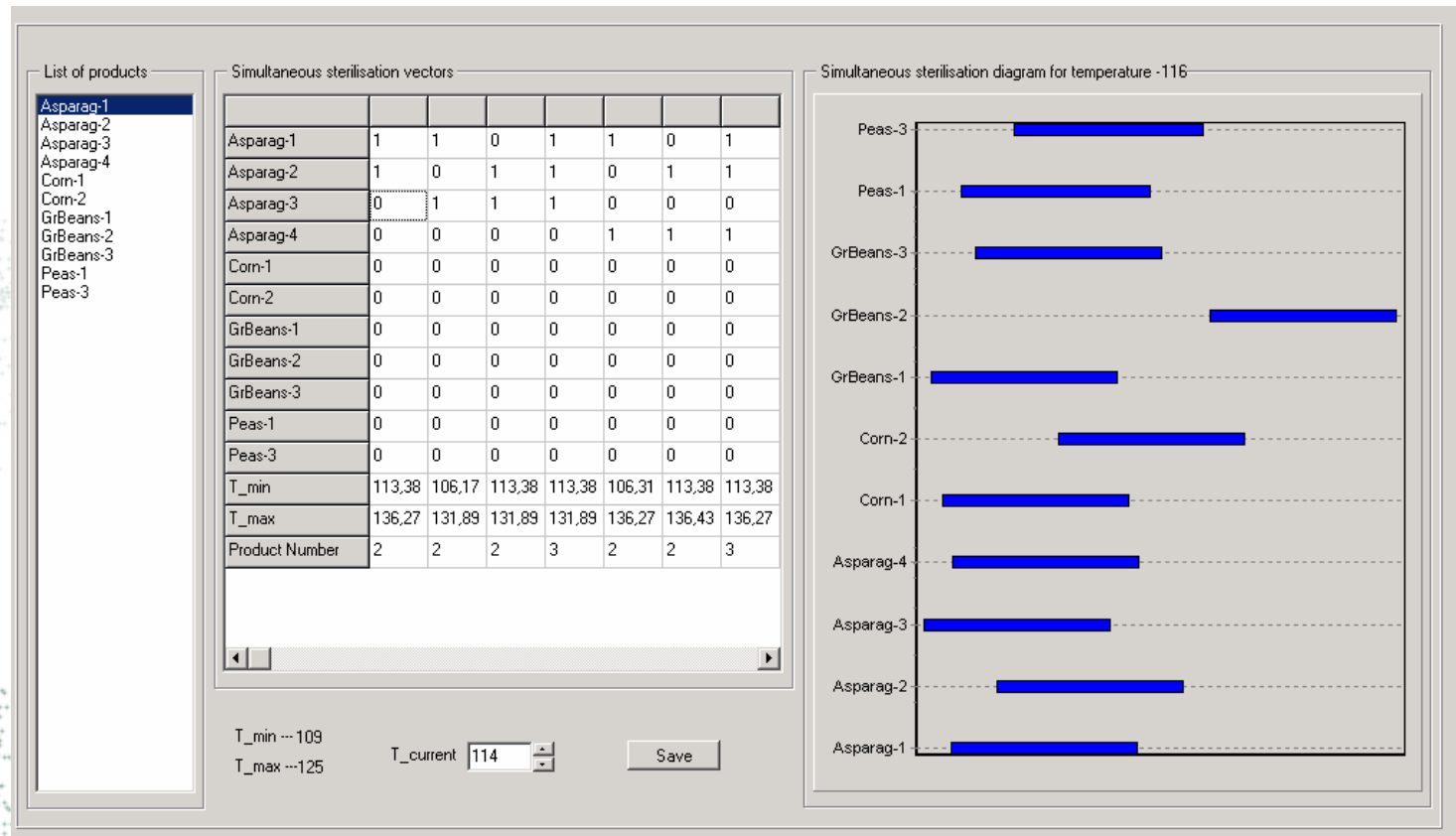
# MIXED INTEGER LINEAR PROGRAMMING (MILP) MODEL

The program has a database of products which can be chosen by the user in any desired combination



# MIXED INTEGER LINEAR PROGRAMMING (MILP) MODEL

After that, the computer program uses all necessary procedures to obtain subsets of simultaneous sterilization vectors.



# MIXED INTEGER LINEAR PROGRAMMING (MILP) MODEL

Calculated subset of sterilization vectors for process temperature 109°C.

Vector	Products																Time (min)
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
1	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	107.63
2	1	0	1	0	0	1	0	1	0	1	0	0	0	0	1	0	110.91
3	1	0	1	1	0	1	0	1	0	1	0	0	0	0	1	0	111.05
4	1	0	1	1	0	1	0	1	0	1	0	0	0	1	1	0	112.56
5	1	0	1	1	0	1	0	1	0	1	0	1	0	1	1	0	114.71
6	1	1	1	1	0	1	0	1	0	1	0	1	0	1	1	0	118.12
7	0	0	0	0	1	0	1	0	1	0	1	0	1	0	0	1	152.25
8	0	1	0	0	1	0	1	0	1	0	0	1	1	0	0	1	144.38
9	1	1	1	1	0	1	0	1	0	1	0	1	0	1	1	1	121.17
10	1	1	1	1	0	1	0	1	1	1	0	1	0	1	1	1	123.36
11	1	1	1	1	0	1	1	1	1	1	0	1	0	1	1	1	127.89
12	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	133.08



# MIXED INTEGER LINEAR PROGRAMMING (MILP) MODEL

For food sterilization problem, the following data is given and generated:

- number of sterilization products:  $n$  ,
- amount for each product:  $a_j, j \in 1:n$  ,
- number of sterilization vectors:  $M$  ,
- set of all possible sterilization vectors:

$$V = \{v^i\}, v_j^i \in \{0,1\}, i \in 1:M, j \in 1:n,$$

- set of sterilization time:  $T = \{t_i\}, i \in 1:M$ ,
- **capacity of each autoclave:  $C$**



# MIXED INTEGER LINEAR PROGRAMMING (MILP) MODEL

---

It should be noted that until the following condition holds:

$$a_j \geq C, \quad \forall j \in 1:n,$$

the algorithm to minimize sterilization time is very simple.

Only non-simultaneous sterilization vectors must be used, because:

- 1) on any sterilization batch, the autoclave (autoclaves) can be completely filled with chosen products and
- 2) any other simultaneous sterilization vector will sterilize a single batch of the same amount of product, but for equal or higher processing time.

# MIXED INTEGER LINEAR PROGRAMMING (MILP) MODEL

The following decision variables are used in the MILP model:

- integer variables:

$$u_i = \begin{cases} 1, & \text{if vector } v^i \in V, \text{ is used for sterilization,} \\ 0, & \text{otherwise.} \end{cases}$$

- continuous variables:  $x_{ij}$ ,  $i \in 1:M$ ,  $j \in 1:n$ ,  
corresponding to an amount of loaded product.

# MIXED INTEGER LINEAR PROGRAMMING (MILP) MODEL

---

The objective function can be written as follows:

$$\sum_{i=1}^M u_i t_i \rightarrow \min,$$

where:  $t_i$  is the time to process the subset of products given by the simultaneous sterilization vector number  $i$ .

It is obvious that the optimal value of a vector of integer variables  $u^*$  equates as a minimum processing time.

# MIXED INTEGER LINEAR PROGRAMMING (MILP) MODEL

Because all given products should be completely sterilized, we therefore have the following constraint.

$$\sum_{i=1}^M x_{ij} = a_j, \quad \forall j \in 1:N.$$

For all chosen simultaneous sterilization vectors  $v^i, i \in 1:M$ , the amount of products loaded in each batch should be less than the given capacity of autoclave  $C$ , and consequently we can write this constraint as:

$$\sum_{j=1}^N x_{ij} \leq u_i C, \quad \forall i \in 1:M.$$

# MIXED INTEGER LINEAR PROGRAMMING (MILP) MODEL

So, the minimization of plant operation time MILP problem can be written as follows:

$$\sum_{i=1}^M u_i t_i \rightarrow \min,$$

subject to:

$$\sum_{i=1}^{\bar{M}} x_{ij} = a_j, \quad \forall j \in 1:N,$$

$$\sum_{j=1}^N x_{ij} \leq u_i C, \quad \forall i \in 1:M.$$



# STUDY CASE

The MILP model developed was tested on 16 products presented in following Table.

Products and can sizes selected for this study

Product	Can size				
	211 × 400	300 × 407	307 × 409	307 × 113	401 × 411
Asparagus	✓	✓	✓	✓	✓
Corn	✓	✓	✓		✓
Green Beans	✓	✓	✓		✓
Peas	✓		✓		✓

Amounts for each given product were generated randomly in accordance with the following condition:

$$a_j \leq C, \forall j \in 1:N.$$

# STUDY CASE

Autoclave with capacity of 10,000 L (available capacity for cans) were chosen, and following amounts for each given product were generated.

Product	1	2	3	4	5	6	7	8
Amount (L)	4300	3400	7600	7200	5500	1900	8100	2500
Product	9	10	11	12	13	14	15	16
Amount (L)	3400	3800	6700	2700	4500	9100	1800	5200

For the given 16 products the isolethal processes were generated (calculated) and by using the software developed the subset  $V$  containing 82 sterilization vectors for temperatures from 109°C to 124°C was obtained.

# STUDY CASE

The obtained problem was solved by using the Mixed Integer Linear Programming Solver of the online Web Service of COIN-OR (COmputational INfrastructure for Operations Research):<http://neos.mcs.anl.gov/neos/solvers/index.html>.

**Results obtained by using the MILP model developed**

Solver name	Computation Time	Objective function value	Non-simultaneous sterilization case
Coin Branch and Cut Solver on NEOS Server version 5.0 using MPS input.	7.38 s.	249.05 min	332.5 min.

# CONCLUSIONS

---

- ✓ The proposed MILP model based on the simultaneous sterilization possibility provides flexibility to optimize battery retort utilization.
- ✓ This procedure is of special relevance for small and medium size canneries that normally work with many different products at the same time.

Furthermore, depending on the practical situation, the MILP model allows tackling other types of optimization problems. For example, the MILP model proposed here can easily be modified to maximize the number of sterilization products for given plant operation time.

# FUTURE TRENDS

---

Our next step is to develop a mathematical model for the optimal scheduling of food-canning plants with autoclaves of different capacities.

In addition, the new models will consider the transient energy consumption.



# OUR GROUP

---

Dr. Sergio Almonacid ([sergio.almonacid@usm.cl](mailto:sergio.almonacid@usm.cl))

Dr. Alik Abakarov ([alik.abakarov@usm.cl](mailto:alik.abakarov@usm.cl))

MSc. Marlene Pinto ([marlene.pinto@usm.cl](mailto:marlene.pinto@usm.cl))

Dr. Ricardo Simpson ([ricardo.simpson@usm.cl](mailto:ricardo.simpson@usm.cl))

**And with the kind and constant support from  
Dr. Arthur Teixeira**



*Muchas Gracias*